

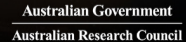
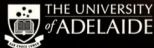
Interferometry part 2 : Noise Budgets

Vaishali Adya, Daniel Brown, Craig Ingram

ECR , December 5 2018

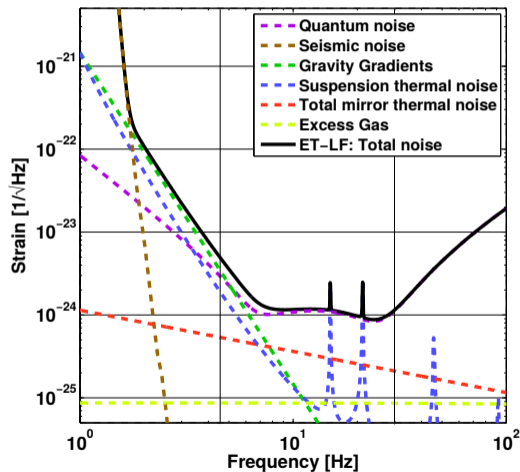


ARC Centre of Excellence for Gravitational Wave Discovery





- Signals and noises
- Time domain or frequency domain?
- Recipe for sensitivity curves
- Do we understand all the noise sources in advanced LIGO?



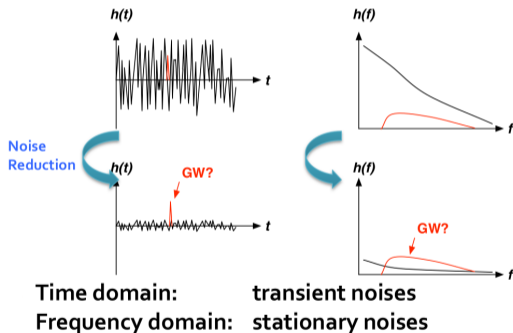


- Signals and noises not well defined
- Mathematically, signal : output from a coherent process—> wanted
- Noise : output from a random (stochastic) process—> unwanted
- Science mode, GW is our signal (can be coherent: CW, inspiral)
- Commissioning mode, try to understand noise, inject noises to see how ifo behaves etc.

Slide courtesy: Andreas Freise, Daniel Brown, Beijing Summer School; Koji Arai : G1601927

Time domain vs frequency domain

Slide idea/courtesy: Andreas Freise, Daniel Brown, Koji Arai



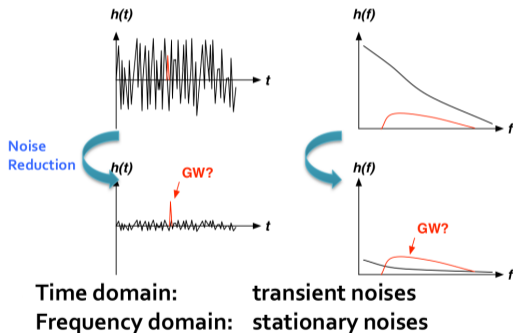


- In certain conditions, noise can be described by its frequency components—>frequency domain
- True only for LTI systems
- Simple and useful because frequency components can be computed separately and added (super-position principle)
- LIGO not always this simple!!
- Transient noise : Raven noise at 90 Hz, oplev glitches, scattered light
- Most of the LIGO modelling done in frequency domain

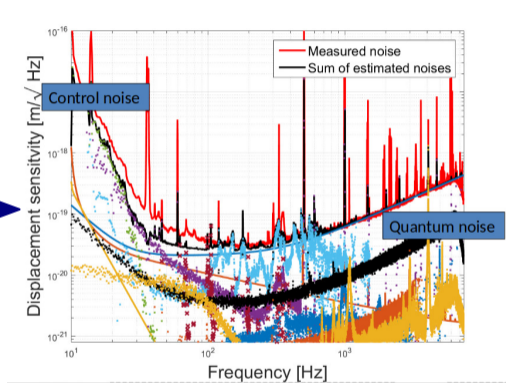
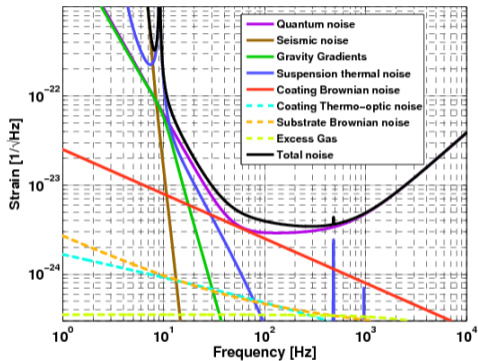
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Time domain vs frequency domain

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Sensitivity curves

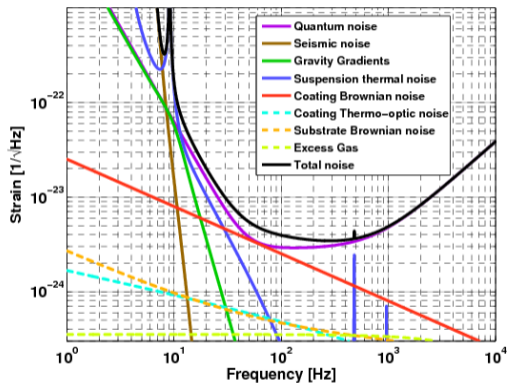


Picture courtesy: S.Dwyer



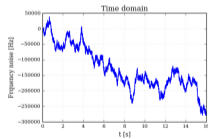
- x-axis : Fourier frequency of noise/signal component (Hz)
- y-axis : h , ASD, gravitational wave strain ($1/\sqrt{\text{Hz}}$)
- Sensitivity curve is essentially noise calibrated to units of strain

Slide courtesy: Andreas Freise, Daniel Brown, Beijing Summer School





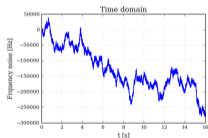
Measured time domain
data (stationary,time invariant)



Brief introduction to noises in frequency domain



Measured time domain
data (stationary,time invariant)



Calculate the Power
Spectral Density (PSD)

For signals with finite duration
and energy, Fourier Transform
and use Parsevals theorem



Commonly use a windowing
function and average the
output spectra

$$S_n(f) = \lim_{T \rightarrow \infty} \frac{2}{T} \left| \int_{-T/2}^{T/2} (n(t) - \bar{n}) e^{-i2\pi f t} dt \right|^2$$

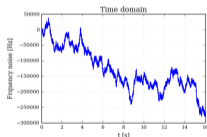
Mean of $n(t)$

Hz²/Hz

Brief introduction to noises in frequency domain



Measured time domain data (stationary, time invariant)



Calculate the Power Spectral Density (PSD)

For signals with finite duration and energy, Fourier Transform and use Parseval's theorem

Commonly use a windowing function and average the output spectra

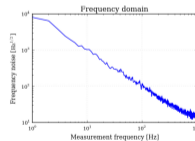
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Mean of $n(t)$

Hz^2/Hz

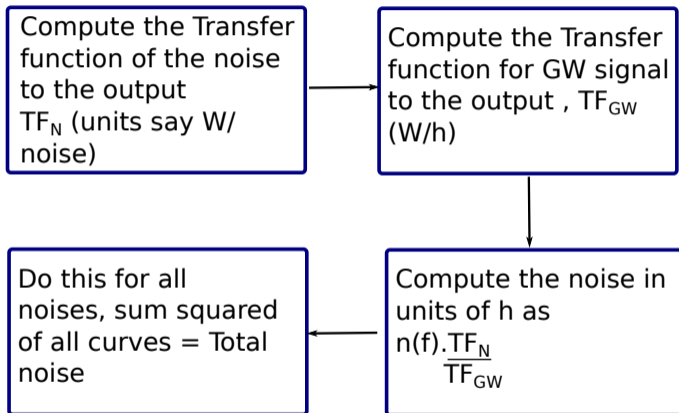
Linearised PSD (Amplitude Spectral Density)

PSD
 $\text{Hz}/\sqrt{\text{Hz}}$



Measure $n(t)$, remove mean of $n(t)$, compute FFT, square, multiply by $2/T$ (depends on FFT function), square root

The recipe to a good noise budget

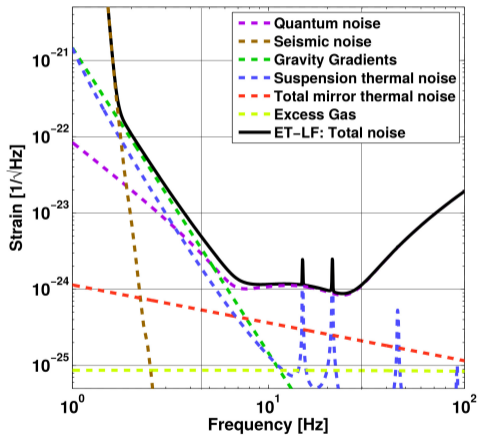


Slide courtesy: adf, ddb

Noise sources in a gravitational wave detector



Seismic noise

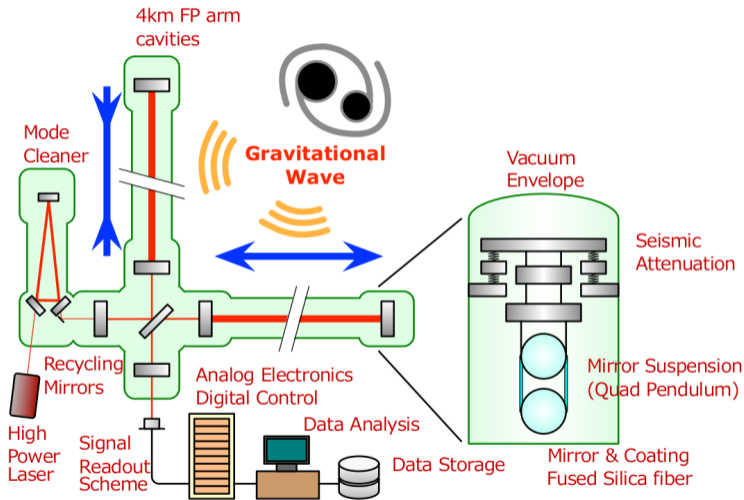


Suspension thermal noise

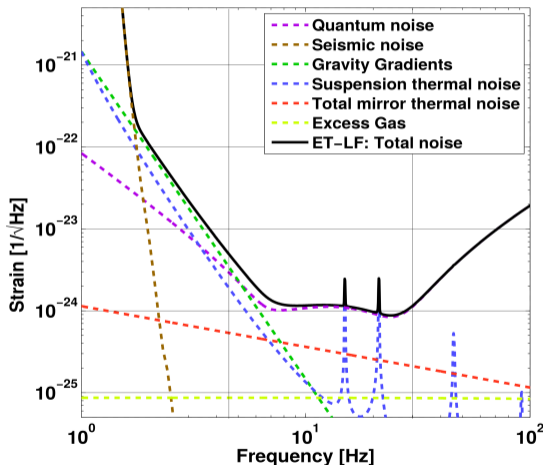
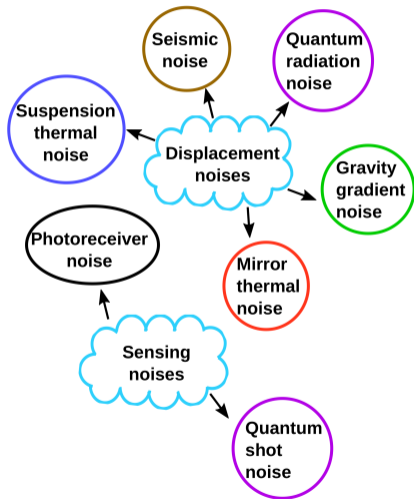
Mirror thermal noise

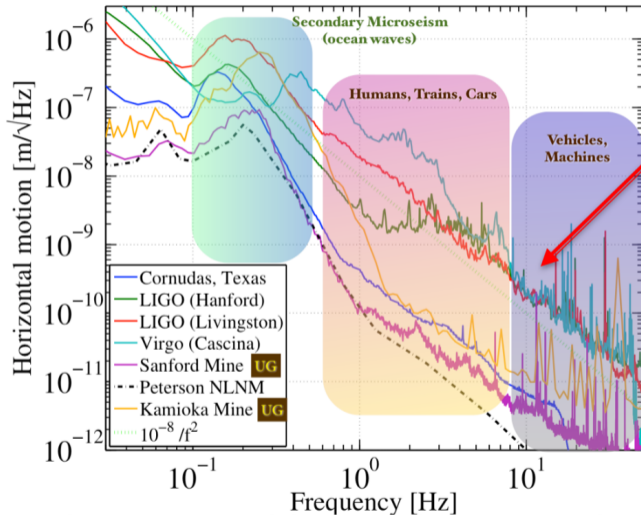
Quantum noise

Noise budget of a gravitational wave detector



Noise budget of a gravitational wave detector





Ground motion
@10Hz
 $10^{-10} \text{m}/\text{rtHz}$

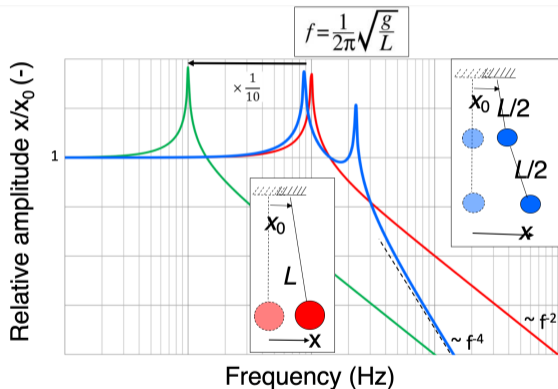
Target
disp. noise
 $10^{-20} \text{m}/\text{rtHz}$



<http://link.aps.org/doi/10.1103/RevModPhys.86.121> (<http://arxiv.org/abs/1305.5188>)

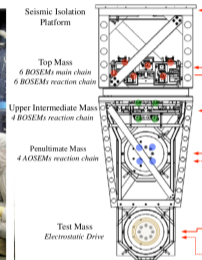
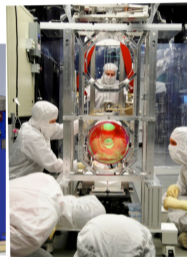
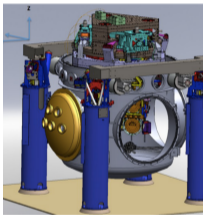


- Very fancy simple harmonic oscillator : provides vibration isolation above its resonant frequency
- Single stage $\frac{1}{f^{2n}}$
- LIGO current isolation system : HAM, suspensions
- Combination of damping(lower peak height-worse isolation), multi stages(steeper isolation curve-more peaks), lower resonant frequency(better isolation-HARD!)



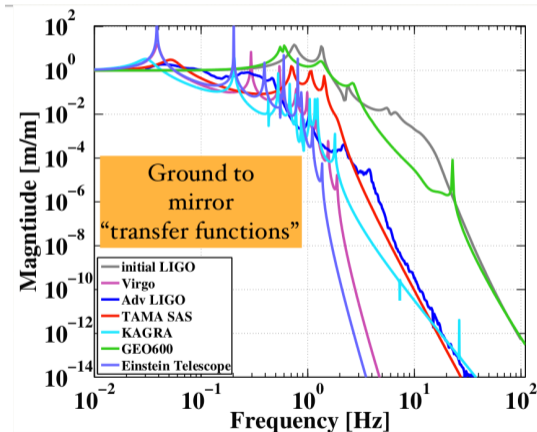


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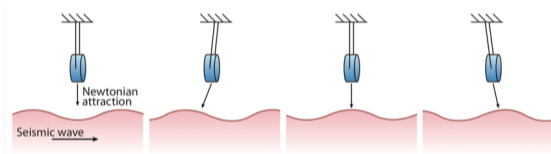


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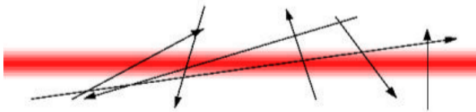
- Seismic noise shakes the mirrors and the environment around the mirror
- This shaking \rightarrow density changes in ground
- Cannot shield mirror from gravity
- Maybe underground detector, shaping the ground around test mass, subtraction of GGN using seismometer array



Picture courtesy : Joris van Heijningen



- Light must travel along the arm without attenuation or degradation
- Refractive index fluctuations in gas cause variations in optical path, phase noise
- Residual gas scatters light out of, then back into, beam; phase noise
- Residual gas pressure fluctuations move mirror; displacement noise
- Contamination: low-loss optics can not tolerate surface 'dirt'; High circulating powers of MWs burns dirt onto optic surface



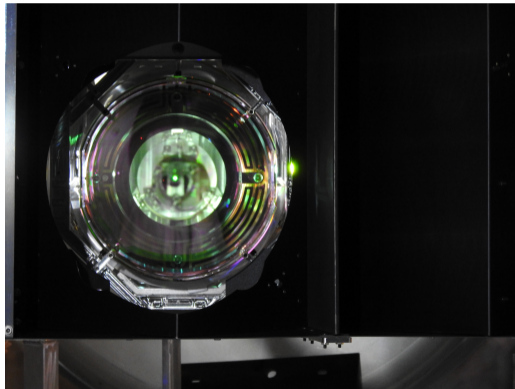
requirement for vacuum in 4 km tubes:

- H_2 at 10^{-6} torr initial, 10^{-9} torr ultimate
- H_2O at 10^{-7} torr initial, 10^{-10} ultimate
- Hydro-, flouorocarbons $< 10^{-10}$ torr
- vacuum system, 1.22 m diameter, $\sim 10,000 \text{ m}^3$
- strict control on in-vacuum components, cleaning



- Scattering of incident beam from surface defects on optics
- This light hits a moving reflector (vibrating tank walls)
- Scattered light recouples to the ifo beam with an arbitrary phase => causes amplitude and phase fluctuation => produces noise in 'h'
- Use of baffles, black glass, use vibration isolation of the scattering object

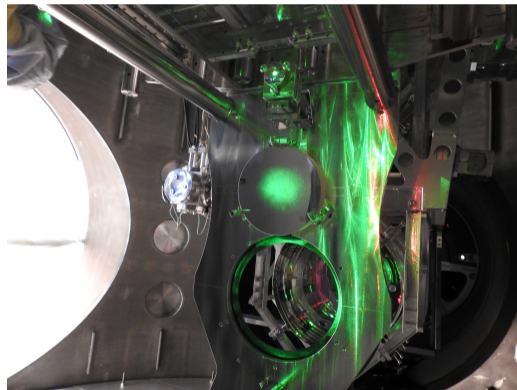
aLIGO Hanford logbook





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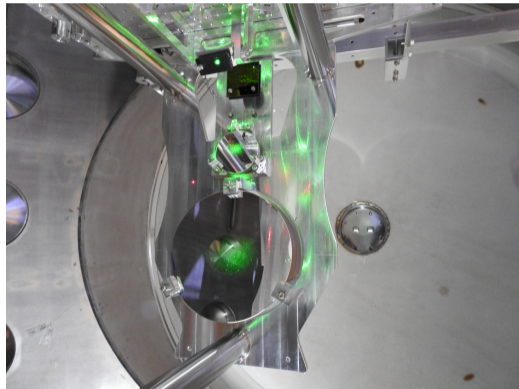
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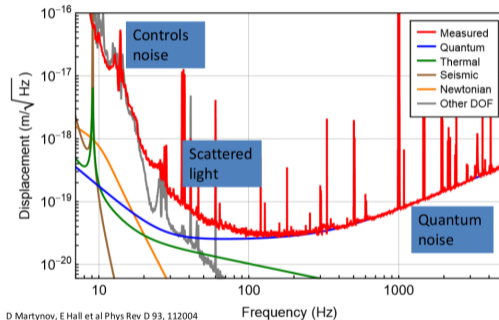
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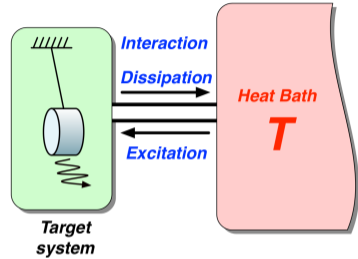


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- Mechanical systems excited by the thermal environment results in physical motions of the tests masses
- Each normal mode of vibration has $k_B T$ of energy; Mirror, many normal modes at discrete frequencies; each will experience thermal excitation.
- Test mass vibrates about its center of mass; but the reflective mirror is on the surface, not the COM, so it introduces displacement noise
- Various noise terms involved: Brownian, thermo-elastic and thermo-refractive noise of each substrate and coating (or coherent combinations of these, such as thermo-optic noise)
- Materials and techniques for very low loss (high Q)



$$S_x(f) = \frac{4k_B T}{\pi^2 f Y} \frac{d}{r_0^2} \left(\frac{Y'}{Y} \phi_{\parallel} + \frac{Y}{Y'} \phi_{\perp} \right)$$

Temperature
Boltzmann constant
Geometrical coating thickness
Loss angle of coating
PSD of displacement
Young's modulus of mirror substrate
laser beam radius
Young's modulus of coating

Harry et al, CQG 19, 897-917, 2002

Picture courtesy : K.Arai; SURF lecture on noises



- Mirrors need to be suspended—>decouple from seismic vibrations
- Suspension wires vibrate (violin modes, stretch/bounce modes), kick the test mass around, introducing an harmonic series of noise lines
- Improve fibre profile (factor of 1.5); increase length of final stage (factor of a few; pushes suspension thermal noise out of detection band); cryogenics(! change of material, requires considerable effort and is complex)

$$x^2(\omega) = \frac{4k_B T \omega_0^2 \phi(\omega)}{\omega m [(\omega_0^2 - \omega^2)^2 + \omega_0^4 \phi^2(\omega)]}$$

PSD of displacement

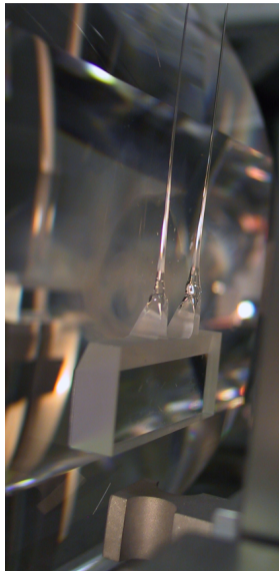
Boltzmann constant

Temperature

Loss angle

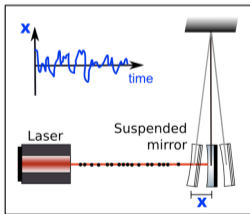
Mirror mass

Resonance frequency





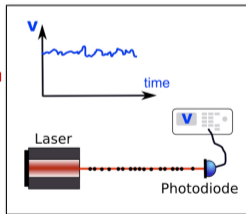
- Quantum noise = shot noise + radiation pressure noise
- Shot noise : due to photon counting statistics
- Quantum radiation pressure noise : Fluctuation of the back action force due to photon number fluctuation
- Also, vacuum fluctuation injected from the dark port=> Differential power fluctuation=> Cause the noise in the GW signal
- Optimal power in the ifo =Trade-off between SN and RPN



photon radiation pressure noise

$$h_{\text{sn}}(f) = \frac{1}{L} \sqrt{\frac{\hbar c \lambda}{2\pi P}}$$
$$h_{\text{rp}}(f) = \frac{1}{m f^2 L} \sqrt{\frac{\hbar P}{2\pi^3 c \lambda}}$$

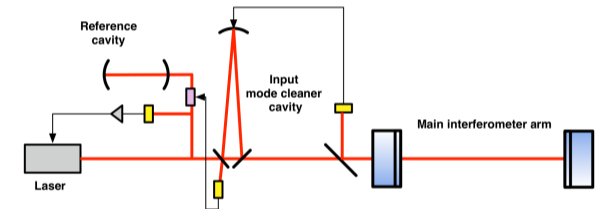
Diagram with labels: wavelength (pointing to λ), optical power (pointing to P), Mirror mass (pointing to m), Arm length (pointing to L).



photon shot noise

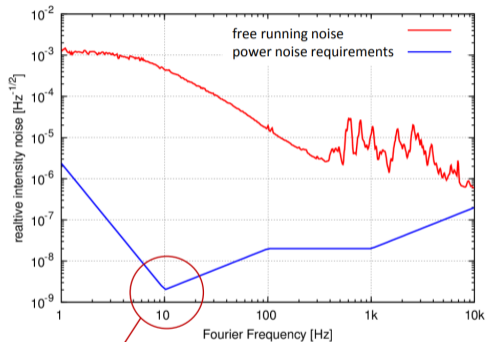


- Laser = yardstick displacement measurement
- Fluctuations in laser frequency manifest as length fluctuations
- Optical phase $\phi = \frac{2\pi\nu L}{c}$
- Change in phase; $d\phi = \frac{2\pi}{c}(Ld\nu + \nu dL)$
- For a 300 THz (1064 nm) beam,
 $d\nu = 10^{-24} * 300 * 10^{12}$ where, $dL/L = 10^{-24}$





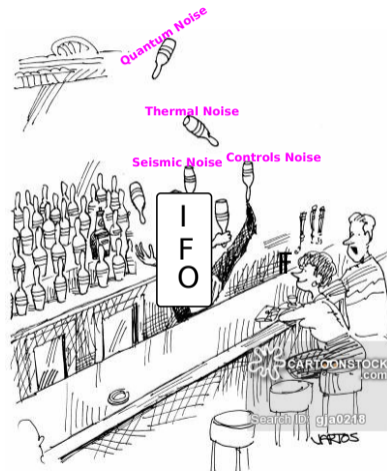
- Requirement in aLIGO for amplitude stabilisation: detecting a photocurrent of 200 mA
- Too much for a single photodiode to handle
- Photodiode array but need new techniques in order to 'power up'



$2 \times 10^{-9} \text{ Hz}^{-1/2}$



- There are such large number of noises which could potentially ruin GW detection
- Careful design / knowledge / experience and Logical, but inspirational trouble shooting needed
- Systematic noise budgeting

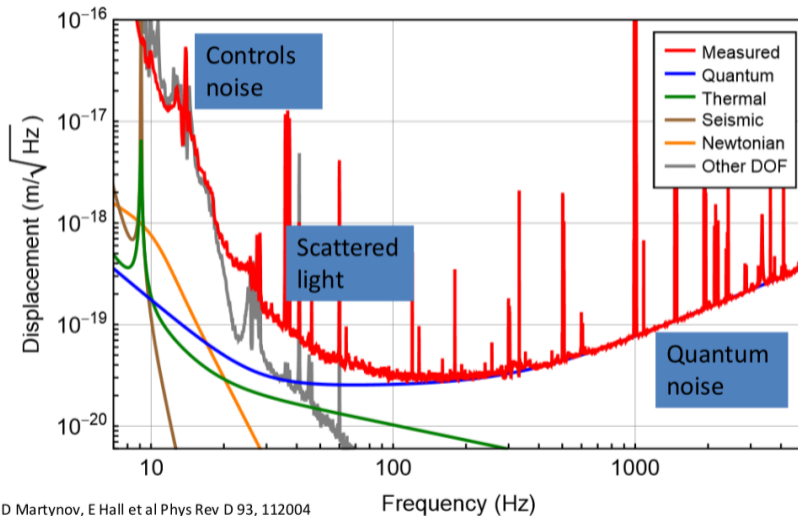


"Yeah he's really good. Too bad he only detected 11 GWs"

He who should not be named.



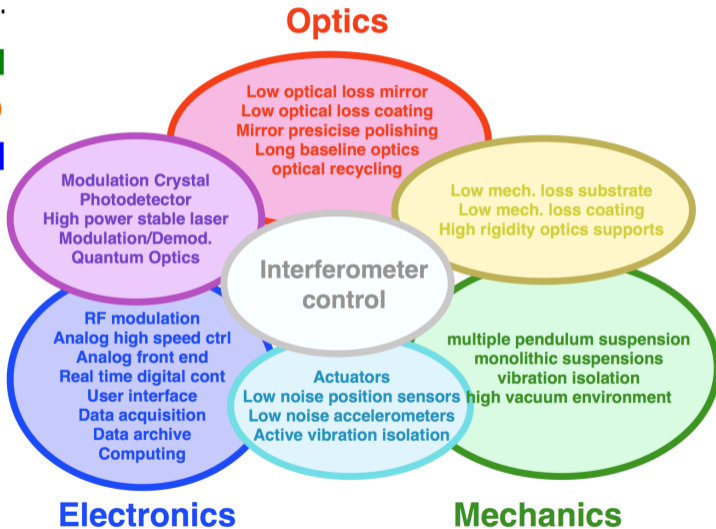
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D Martynov, E Hall et al Phys Rev D 93, 112004



- 3'
- M
- O
- EI



Build your own detector!

